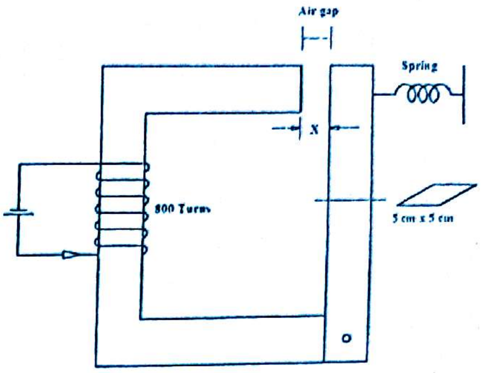
3/5/2017 م.خالد فيرلو

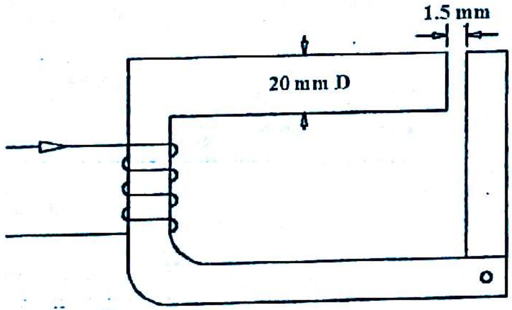
University of Tripoli - Faculty of Engineering
Electrical and Electronic Engineering Department

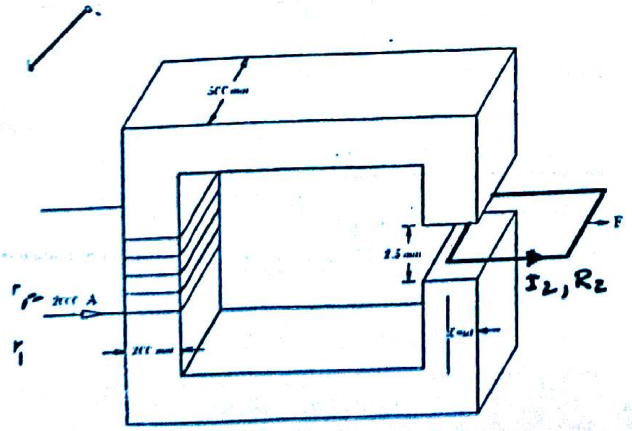
EE352 Electro-Mechanical Energy Conversion Spring 2013 Second Exam Time:1 H

1. **An electromagnetic relay has an exciting coil of 800 turns. The core has a cross sectional area of (5 cm x 5 cm). Neglecting reluctance of the magnetic circuit and fringing.**
   - **a. Find the coil inductance if the air gap length is 0.5 cm. (2 marks)**
   - **b. Find the field energy stored for a coil current of 1.25 Amp. (2 marks)**



1. The relay shown below operates at a constant voltage of 50 V. the air gap is initially 2.0 mm long. When the relay is energized, the armature closes passing the point at which the air gap is 1.5 mm long with a velocity of 1.0 m/s. if the relay takes a current of 1000 mA when closed, make an estimate of the current when the air gap is 1.5 m. hence estimate the power component and the force components and the force on the armature.





2. The device shown in fig(2) has the following dimensions lg = 2.5 mm, b = 200 mm, l = 500 mm. the excitation coil carries a current of 2000 A and has a resistance of 0.005 Ω. The closed coil travels with a velocity of 5.0 m/s and has a resistance of 0.005 Ω. Calculate the force of interaction experienced by the closed coil.

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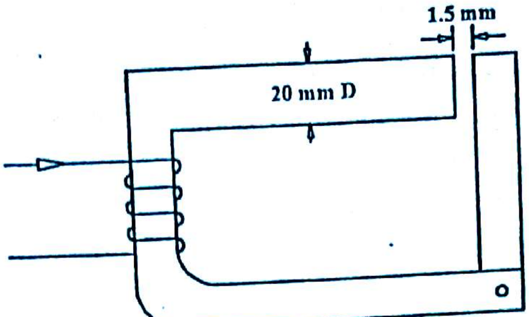


'ssume that the flux does not change appriciably

$IN = \Phi \mathcal{R}$ From which $\frac{I_1}{I_2} = \frac{l_1}{l_2}$

$I_2 = 75 \times 10^{-3}$ A $= 75$ mA

$V = ir + L\frac{di}{dt} + i\frac{dL}{dt}$

$= i_r r + L\frac{di}{dt} + i\frac{dL}{dx} * \frac{dx}{dt}$ but $L = \frac{N^2 \mu_0 A}{l_g}$

$L = 10.25$ H

$\frac{dL}{dx} = \frac{N^2 \mu_0 A}{l_g^2} = -\frac{L}{l_g} = \frac{10.25}{1.5 \times 10^{-3}} = -688$ H/m

$v = \frac{dx}{dt} = -1.0$ m/s

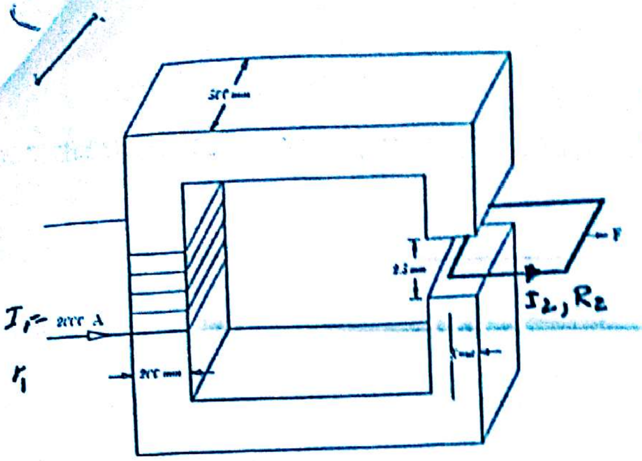
Substituting in the equation of the voltage

$v = 50 = \frac{75}{1000} \times \frac{50}{0.1} + 10.25 \times \frac{di}{dt} + \frac{75}{1000} \times -688 \times -1$

$\frac{di}{dt} = 3.8$ A/s

Power from the source $= vi = 50 \times 75 \times 10^{-3} = 3.75$ W

[illegible]



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$I_1 = 2000\,A$, $r = 0.005\,\Omega$

The initial flux density in the air gap

$$B_1 = \mu_0 H_1 = \mu_0 \frac{I_1}{l_g} = \frac{4\pi\times10^{-7}\times2000}{2.5\times10^{-3}} = 1.0 \quad wb/m^2$$

The flux density B_2 is given by.

$$B_2 = \mu_0 H_2 = \mu_0 \frac{(I_1+I_2)}{l_g}$$

$\Phi_2 = B_2 \, l x = B_2 \, l u t$ where u = velocity.

$$\frac{d\Phi_2}{dt} = B_2 \, l u = E_2$$

$$I_2 = -\frac{E_2}{R_2} = -\frac{B_2 l u}{R_2} = -(I_1+I_2)\overset{K}{\boxed{\frac{\mu_0 l u}{l_g R_2}}}$$

$K = 0.25 =$

$$I_2 = -(I_1+I_2)K$$

$$I_2 = -\frac{K}{1+K} \cdot I_1 = -400\,A$$

$$B_2 = \frac{(I_1+I_2)\mu_0}{l_g} = 0.8\,T$$

$\boxed{8}$

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EE 352

Q1

The magnetic circuit of Fig (1) consists of a single coil stator and oval rotor. Because the airgap is non uniform, the coil ~~induct~~ inductance varies with rotor angular position, measured between the magnetic axis of the stator coil and the major axis of the rotor.

$$L(\theta) = L_0 + L_2 \cos 2\theta .$$

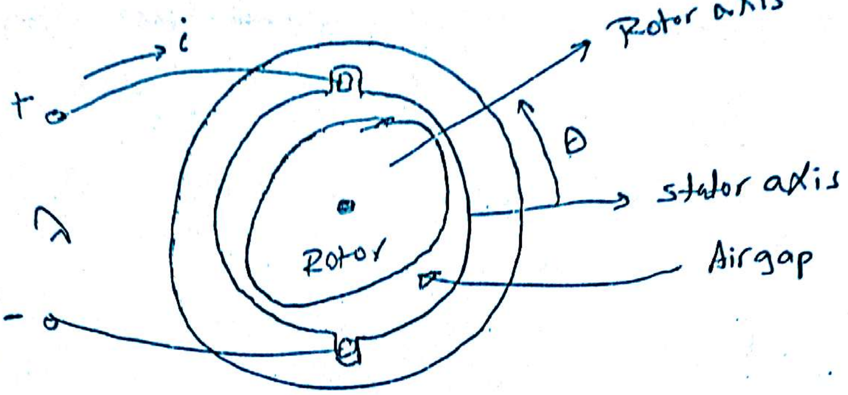


Fig (1)

Where $L_0 = 10.6$ mH and $L_2 = 2.7$ mH. Note the second-harmonic variation of inductance with rotor angle θ. This is consistent with the fact that the inductance is unchanged if the rotor is rotated through an angle of 180°.

Find the torque as function of θ for a coil current of 2A.

$$\because W_f = W_f' = \frac{1}{2} L(\theta)\, i^2$$

$$\therefore T_{fld} = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} = \frac{i^2}{2}\left(-2\sin(2\theta) L_2\right) \quad \because i = 2A.$$

$$\boxed{\therefore T_{fld} = -1.08\times 10^{-2} \sin 2\theta \ \text{N.m}}$$

Q2 The inductance of a coil on a magnetic circuit similar to hat of Fig (1) is found to vary with rotor position as

$$L(\theta) = L_0 + L_2 \cos 2\theta + L_4 \sin 4\theta$$

here $L_0 = 25.4 mH$, $L_2 = 8.3 mH$ and $L_4 = 1.8 mH$.

a) Find the Torque as a function of θ for a winding current of 3.5A.

b) Find a rotor position θ_{max} that produces the largest negative torque.

Solution

a] $W_{fld} = \frac{1}{2} L(\theta) i^2$

$\therefore T_{fld} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} = \frac{1}{2}(3.5)^2 \frac{d}{d\theta}(L_0 + L_2 \cos 2\theta + L_4 \sin 4\theta)$

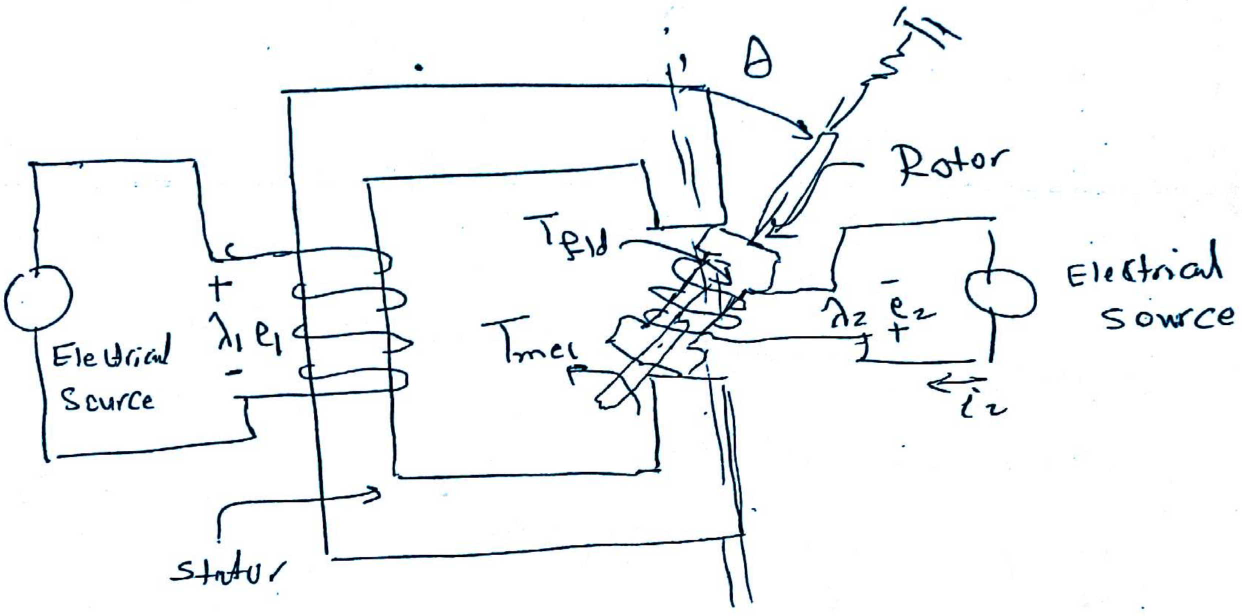
$\therefore T_{fld} = \frac{1}{2}(3.5)^2 \left[- 8.3 \times 10^{-3} \sin 2\theta \times 2 + 4 \times 1.8 \times 10^{-3} \cos 4\theta \right]$

$$\boxed{T_{fld} = -0.1017 \sin 2\theta + 0.0441 \cos 4\theta \ N.m}$$

b] The largest torque occurs when $\theta = 45°$ and $\theta = 225°$.

Because if we subsitute by $\theta = 45°$ & $22.5°$ ~~we~~ we find the ~~max~~ largest negative torque.

Q3 In the system shown in Fig (2) the inductances in henerys are given as $L_{11} = (3 + \cos 2\theta) \times 10^{-3}$, $L_{12} = 0.3 \cos\theta$ $L_{22} = 30 + 10 \cos 2\theta$. Find and plot the torque $T_{fld}(\theta)$ for current $i_1 = 0.8A$ and $i_2 = 0.01 A$.



Solution.

$$W_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + i_1 i_2 L_{12}.$$

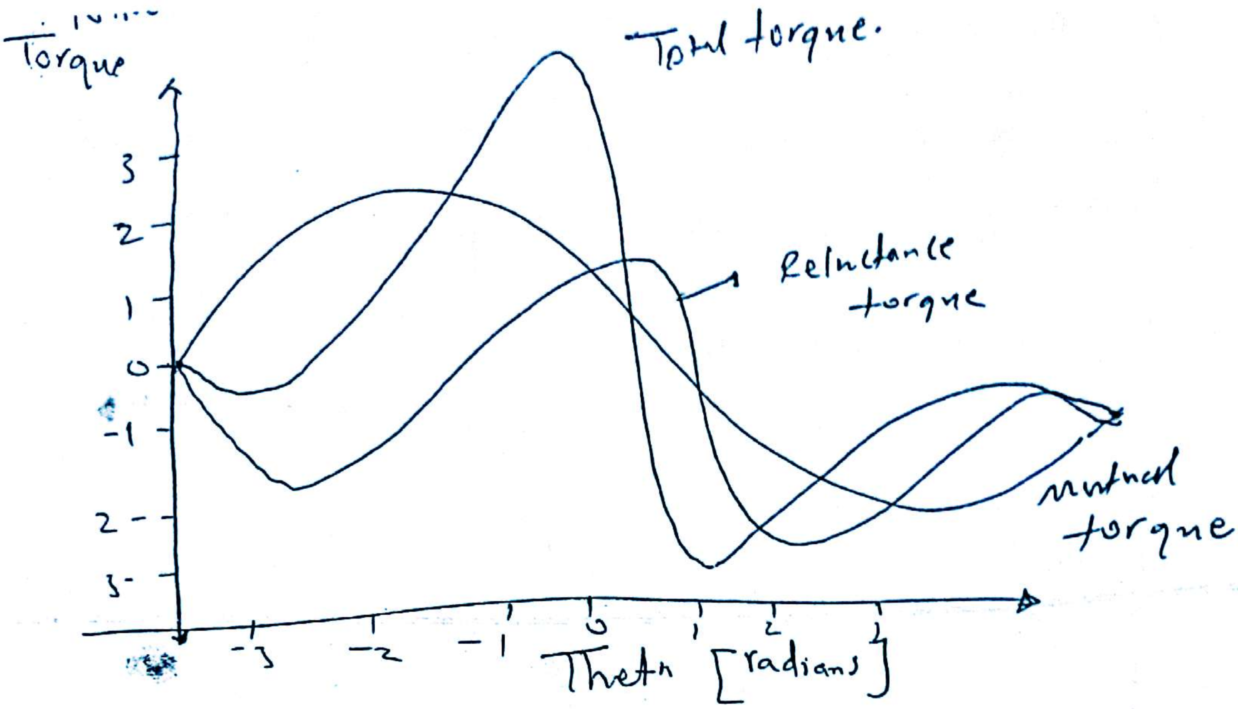
$$T_{fld} = \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}.$$

$$T_{fld} = \frac{i_1^2}{2}(-2 \times 10^{-3}) \sin 2\theta + \frac{i_2^2}{2}(-20 \sin 2\theta) - i_1 i_2 (0.3) \sin\theta$$

For $i_1 = 0.8A$. $i_2 = 0.01 A$.

$$T_{fld} = \frac{(0.8)^2}{2}(-2 \times 10^{-3} \sin 2\theta) + \frac{(0.01)^2}{2}(-20 \sin 2\theta) - (0.01)(0.8)(0.3)\mathrm{Si}$$

$$T_{fld} = -1.64 \times 10^{-3} \sin 2\theta - 2.4 \times 10^{-3} \sin\theta.$$



Q4 For Fig(2) Find an expression for torque of a symmetrical two winding system whose inductances vary as

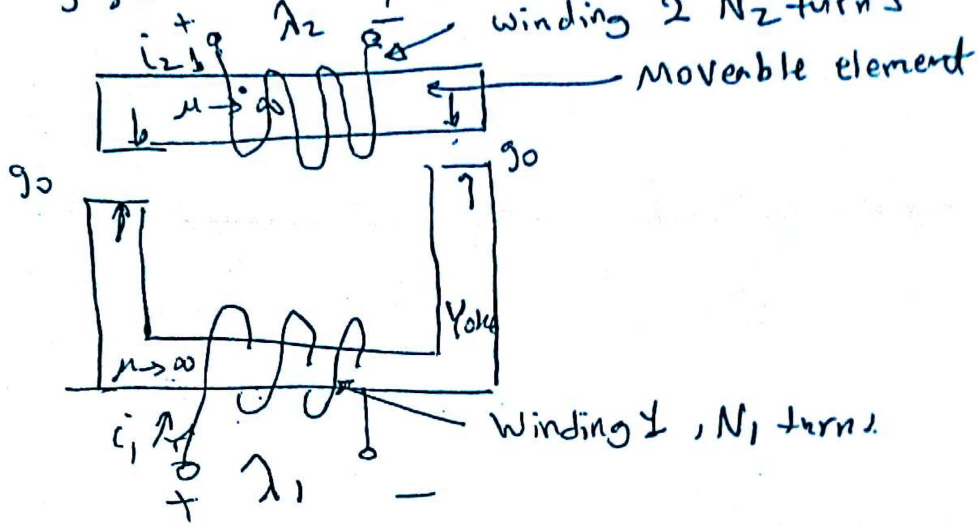
$$L_{11} = L_{22} = 0.8 + 0.27\cos 4\theta.$$

$$L_{12} = 0.65\cos 2\theta.$$

For the condition that $i_1 = -i_2 = 0.37A$.

$$T_{fld} = -0.296\sin 4\theta + 0.178\sin 2\theta.$$

Q_5 The two-winding magnetic circuit of Fig(3) has a winding on a fixed yoke and a second winding on a movable element. The moveable element is constrained to motion such that the lengths of both airgaps remain equal.



Fig(3)

ⓐ Find the self inductances windings 1 and 2 in terms of the core dimensions and the number of turns.

b] Find the mutual inductance between the two windings.

c] Calculate the Co-energy $W'_{fld}(i_1, i_2)$

d) Find an expression for the force acting on the moveable element as a function of the winding currents.

a)

$$L_{11} = \frac{N_1^2}{R(x)} = \frac{\mu_0 N_1^2 A}{2g_0}, \quad L_{22} = \frac{\mu_0 N_2^2 A}{2g_0}$$

b $L_{12} = \dfrac{\mu_0 N_1 N_2 A}{2 g_0}$.

c $W'_{fld} = \dfrac{1}{2} L_{11} i_1^2 + \dfrac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2$

$= \dfrac{\mu_0 A}{4 g_0} (N_1 i_1 + N_2 i_2)^2$ ✓

d $f_{fld} = \dfrac{\partial W'_f}{\partial g_0}\Big|_{i_1, i_2} = -\dfrac{\mu_0 A}{4 g_0^2} (N_1 i_1 + N_2 i_2)^2$ φ

Q6 Two coils are mounted on a stator and the other on a rotor, have self - and mutual inductors of $L_{11} = 3.5\,mH$, L_{22} $1.8\,mH$, $L_{12} = 2.1\cos\theta\,mH$.

where θ is the angle between the axes of the coils. The Coils are Connected in series and carry a current

$i = \sqrt{2}\, I \sin\omega t$

(a) Derive an expression for the instantaneous torque on the rotor as function of the angular position θ.

(b) Find an expression for the time-averaged torque T_{avg} as a function of θ.

(c) Compute the numerical Value of T_{avg} for $I = 10A$ and $\theta = 90°$

Solution

(a) $\therefore W'_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} l_{22} i_2^2 + i_1 i_2 L_{12}.$

$$\boxed{T = \frac{\partial W'_{fld}}{\partial \theta} = -4.2\times 10^{-3} I^2 \sin\theta \sin^2\omega t}$$

$\omega t = 90^\circ$

$\sin^2\omega t = \frac{1 - \cos 2\theta}{2} = \left(\frac{1+0}{2}\right),$

(b) $T_{fld} = -2.1\, I^2 \sin\theta$ N.m

(c) $\because I = 10A,\ \theta = 90^\circ$

$\therefore$ $T_{fld} = -0.21$ N.m.

Q7 Two windings, one mounted on a stator and the other on a rotor, have self and mutual inductances of

$$L_{11} = 4.5, \quad L_{22} = 2.5H, \quad L_{12} = 2.8\cos\theta \, H.$$

where θ is the angle between the axes of the winding The resistances of the windings may be neglected. Winding is short-circuited, and the current winding 1 as a function of time is $i_1 = 10\sin\omega t$ A.

a) Derive an expression for the numerical value in N.m of instantanous torque on the rotor in terms of the angle θ.

b) Compute the time averaged torque in N.m when $\theta = 45^\circ$

Solution

a) $T_{fld} = i_1 i_2 \frac{dL_{12}}{d\theta} = -2.8 i_1 i_2 \sin\theta$ N.m.

∵ $\lambda_2 = 0 \Rightarrow i_2 = -\left(\frac{L_{12}}{L_{22}}\right) i_1 = -1.12 i_1 \cos\theta$

∵ $i_1 = 10 \sin\omega t$

$T_{fld} = -3.14 i_1^2 \sin\theta \cos\theta = -314\sin^2(\omega t)\sin\theta\cos\theta$

$T_{fld} = -78.5(1-\cos(2\omega t))\sin 2\theta$ N.m.

b] $T_{fld} = -78.5$ N.m